More examples of direct and indirect proofs

Andrew Roberts

4th November 2004

Direct proofs

1. If n is even, then n^2 is even.

If n is even, then $n = 2k$ for some integer k.

$$\begin{aligned} n^2 &= (2k)^2 \\ &= (2k)(2k) \\ &= 2(2k^2) \\ &= 2l \qquad l \in \mathbb{Z} \end{aligned} \tag{1}$$

Therefore, n^2 is even.

2. If n is odd, then n^2 is odd [*from your lecture notes*].

If n is odd, then $n = 2k + 1$ for some integer k.

$$\begin{aligned} n^2 &= (2k+1)^2 \\ &= 4k^2 + 4k + 1 \\ &= 2(2k^2 + 2k) + 1 \\ &= 2l + 1 \qquad l \in \mathbb{Z} \end{aligned} \tag{2}$$

Therefore, n^2 is odd.

Indirect proofs

1. If n^2 is even, then n is even.

Assume n^2 be even*. Suppose that n is odd. Therefore, $n = 2k+1$ for some integer k.

$$\begin{aligned} n^2 &= (2k+1)^2 \\ &= 4k^2 + 4k + 1 \\ &= 2(2k^2 + 2k) + 1 \\ &= 2l + 1 \qquad l \in \mathbb{Z} \end{aligned} \tag{3}$$

Therefore, n^2 is odd. This contradicts our assumption(*), so n must be even.

2. If $5n + 2$ is odd, then n is also odd. [*from your lecture notes*]

Let $5n+2$ be odd*. Suppose that n is even. Therefore, $n = 2k$ for some integer k.

$$\begin{aligned} 5n + 2 &= 5(2k) + 2 \\ &= 10k + 2 \\ &= 2(5k + 1) \\ &= 2l \qquad l \in \mathbb{Z} \end{aligned} \tag{4}$$

Therefore, $5n+2$ is even. This contradicts our assumption(*), so n must be odd.