Notes on Validity and Fallacy

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1 Introduction

To summarise from the lecture notes:

1. Rules of inference are a mechanism by which one can draw conclusions from a set of hypotheses.
2. Due to the emphasis of correctness, when these rules are written as propositions, they are tautologies.
3. An argument must be built from correct rules of inference to be valid. A valid proof requires that every argument and its premises are all true.
4. A fallacy occurs when an inccorect argument has been used. This arises when arguments are in fact based on a contingency, rather than tautology.

2 Fallacy

2.1 Fallacy of affiming the conclusion

This fallacy is based on the following proposition:

$$((p \to q) \wedge q) \to p$$

This equivalent to saying that a converse implication, $q \to p$, is not the same its direct implication, $p \to q$.

Example: If x is a real number such that $x > 1$ then $x^2 > 1$. Suppose that $x^2 > 1$. Then $x > 1$.

Let:

$$\begin{aligned} p &: x > 1 \\ q &: x^2 > 1 \end{aligned}$$

The example is stating:

$$
\begin{array}{rl}
 & p \to q \\
 & q \\
\hline
\therefore & q \to p
\end{array}
$$

This conforms to the proposition stated earlier, which means that this argument is invalid: *a fallacy of affirming the conclusion.* To give actual examples that show the weakness in the argument, when $x < -1$, then $x^2 > 1$.

2.2 Fallacy of denying the hypothesis

This fallacy is based on the following proposition:

$$((p \to q) \wedge \neg p) \to \neg q$$

This fallacy is like saying that $p \to q$ is equivalent to $\neg p \to \neg q$, which it clearly is not the case.

Example: If x is a real number such that $x > 2$then $x^2 > 4$. Suppose that $x \leq 2$. Then $x^2 \leq 4$.

Let:

$$
\begin{array}{rcl}
p & : & x > 2 \\
q & : & x^2 > 4
\end{array}
$$

The example is stating:

$$
\begin{array}{rl}
 & p \to q \\
 & \neg p \\
\hline
\therefore & \neg p \to \neg q
\end{array}
$$

It should now be clear that such arguments lead to the *fallacy of denying the hypothesis*. For example, as when $x < -2$ then $\neg q$ is no longer satisfied, and in fact becomes $x^2 > 4$.

2.3 Circular reasoning

This should be an obvious fallacy to spot, as it occurs when at a point during a proof, you make the assumption that the hypothesis being proved is actually true (think catch-22). This makes the proof meaningless, and is therefore incorrect.

Example[1]**:**

Prove: if n^2 is not divisible by 3, then n is not divisible by 3.

Now, consider the following proof:

> Since n^2 is not divisible by 3, hence n^2 cannot be equal to $3k$ for some integer k. Thus, if n cannot be equal to $3l$ for some integer l, therefore, n is not divisible by 3.

If you read carefully, you should notice that the argument 'if n cannot be equal to $3l$' is equivalent to the statement being proved, which makes it a circular argument, and therefore an illegal proof.

[1]Example courtesy of lecture notes by Wojciech Szpankowski (at Purdue University, USA)