Examples of matrix proofs

12th February 2005

The following proofs are for theorem 1.1, on page 6, in your notes. I believe that the proofs themselves are a clear enough guide as how to perform them yourselves. Remember, do one step at a time, and remember to justify it.

Theorem 1.1 *Let A, B and C be matrices with the same size, and let k and k' be scalars. The the following hold:*

(i) $(A + B) + C = A + (B + C)$

(ii) $A + B = B + A$

(iii) $A + 0 = A$

(iv) $A + (-A) = 0$

(v) $k(A + B) = kA + kB$

(vi) $(k + k')A = kA + k'A$

(vii) $(kk')A = k(k'A)$

(viii) $1A = A$

Proof: Let $A = [a_{ij}]$, $B = [b_{ij}]$, $C = [c_{ij}]$ be *m*x*n* matrices. Let k and k' be scalars.

Proof of (i):

Should be obvious. Give it a go!

Proof of (ii):

$$
\begin{aligned}
A + B &= [a_{ij}] + [b_{ij}] && \text{(by notation)} \\
&= [a_{ij} + b_{ij}] && \text{(by definition of matrix sum)} \\
&= [b_{ij} + a_{ij}] && \text{(property of numbers)} \\
&= [b_{ij}] + [a_{ij}] && \text{(by definition of matrix sum)} \\
&= B + A && \text{(by notation)}
\end{aligned}
$$

Proof of (iii):

$$\begin{aligned} A+0 &= [a_{ij}]+0 && \text{(by notation)} \\ &= [a_{ij}+0] && \text{(by definition of matrix sum)} \\ &= [a_{ij}] && \text{(property of numbers)} \\ &= A && \text{(by notation)} \end{aligned}$$

Proof of (iv):

This one is on your coursework!

Proof of (v):

$$\begin{aligned} k(A+B) &= k([a_{ij}]+[b_{ij}]) && \text{(by notation)} \\ &= k[a_{ij}+b_{ij}] && \text{(by definition of matrix sum)} \\ &= [k(a_{ij}+b_{ij})] && \text{(by definition of product of a matrix by scalar)} \\ &= [ka_{ij}+kb_{ij}] && \text{(property of numbers)} \\ &= [ka_{ij}]+[kb_{ij}] && \text{(by definition of matrix sum)} \\ &= k[a_{ij}]+k[b_{ij}] && \text{(by definition of product of a matrix by scalar)} \\ &= kA+kB && \text{(by notation)} \end{aligned}$$

Proof of (vi):

$$\begin{aligned} (k+k')A &= (k+k')[a_{ij}]) && \text{(by notation)} \\ &= [(k+k')a_{ij}] && \text{(by definition of product of a matrix by scalar)} \\ &= [ka_{ij}+k'a_{ij}] && \text{(property of numbers)} \\ &= [ka_{ij}]+[k'a_{ij}] && \text{(by definition of matrix sum)} \\ &= k[a_{ij}]+k'[a_{ij}] && \text{(by definition of product of a matrix by scalar)} \\ &= kA+k'A && \text{(by notation)} \end{aligned}$$

Proof of (vii):

$$\begin{aligned} (kk')A &= (kk')[a_{ij}]) && \text{(by notation)} \\ &= [(kk')a_{ij}] && \text{(by definition of product of a matrix by scalar)} \\ &= [k(k'a_{ij})] && \text{(property of numbers)} \\ &= k[k'a_{ij}] && \text{(by definition of product of a matrix by scalar)} \\ &= k(k'[a_{ij}]) && \text{(by definition of product of a matrix by scalar)} \\ &= k(k'A) && \text{(by notation)} \end{aligned}$$

Proof of (viii):

Easy one! Have some practise.