

# More Vector Examples

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Here are a few examples of putting the principles of vector addition, difference and scalar product into action.

**Example 1.** Given  $x \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \end{bmatrix}$ . Find  $x$  and  $y$ .

Firstly, apply the rules of scalar products. This gives:

$$\begin{bmatrix} 2x \\ x \end{bmatrix} + \begin{bmatrix} y \\ -2y \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \end{bmatrix}$$

Next, perform the addition:

$$\begin{bmatrix} 2x+y \\ x-2y \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \end{bmatrix}$$

You can now consider each row at a time.

- ①  $2x + y = 1$
- ②  $x - 2y = -7$

If you recall from your GCSE days, you should instantly recognise this as a simultaneous equation. The aim is to deduce the values of  $x$  and  $y$ . This can be achieved by taking one of the equations, and manipulating so that it equals either one of the terms. Once done, you can substitute in to the remaining equation, which leaves one unknown term, and that can be calculated easily.

Taking equation ①, it is very simple to make it equal to  $y$ :

$$2x + y = 1 \Rightarrow y = 1 - 2x$$

Now we know what  $y$  equals, substitute this value for  $y$  in equation ②.

$$\begin{aligned} x - 2y = -7 &\Rightarrow x - 2(1 - 2x) = -7 \\ &\Rightarrow x - (2 - 4x) = -7 \\ &\Rightarrow 5x - 2 = -7 \end{aligned}$$

Now solve for  $x$ .

$$\begin{aligned} 5x - 2 = -7 &\Rightarrow 5x = -5 \\ &\Rightarrow x = \frac{-5}{5} \\ &\Rightarrow x = -1 \end{aligned}$$

Now that we know the actual value for  $x$ , it is only a quick step to calculate the value for  $y$ . Just simply substitute the value of  $x$  in either of the original equations, leaving  $y$  as the only unknown. Then solve for  $y$ .

For example, substituting  $x$  for  $-1$  in equation ①:

$$\begin{aligned}2x + y = 1 &\Rightarrow 2(-1) + y = 1 \\ &\Rightarrow -2 + y = 1 \\ &\Rightarrow y = 1 + 2 \\ &\Rightarrow y = 3\end{aligned}$$

We can now conclude that  $x = -1$  and  $y = 3$ .

**Example 2.** Given  $x \begin{bmatrix} -1 \\ 4 \end{bmatrix} - y \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 19 \end{bmatrix}$ . Find  $x$  and  $y$ .

$$x \begin{bmatrix} -1 \\ 2 \end{bmatrix} - y \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -x - 2y \\ 4x - y \end{bmatrix} = \begin{bmatrix} 2 \\ 19 \end{bmatrix}$$

$$\textcircled{1} \quad -x - 2y = 2$$

$$\textcircled{2} \quad 4x - y = 19$$

Arbitrarily picking one of the equations, ②, I'd say it is easier to transpose when equal to  $y$ :

$$\begin{aligned}4x - y = 19 &\Rightarrow 4x = 19 + y \\ &\Rightarrow 4x - 19 = y\end{aligned}$$

Now substitute into 1:

$$\begin{aligned}-x - 2y = 2 &\Rightarrow -x - 2(4x - 19) = 2 \\ &\Rightarrow -x - (8x - 38) = 2 \\ &\Rightarrow -x - 8x + 38 = 2 \\ &\Rightarrow 38 - 9x = 2 \\ &\Rightarrow 36 = 9x \\ &\Rightarrow \frac{36}{9} = x \\ &\Rightarrow x = 4\end{aligned}$$

With this result, substitute into one of the original equations, I shall choose ②, and solve for  $y$ .

$$\begin{aligned}4x - y = 19 &\Rightarrow 4(4) - y = 19 \\ &\Rightarrow 16 - y = 19 \\ &\Rightarrow y = 16 - 19 \\ &\Rightarrow y = -3\end{aligned}$$

Therefore,  $x = 4$  and  $y = -3$ .